Immigration, Labor Dynamics, and Fiscal Sustainability

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Abstract

A descriptive analysis of OCDE data suggests that a positive relationship exists between the population share of seniors and public debt as percentage of GDP. This fact points to the relevance of population structure for public finance management. Because Immigration can deeply modify countries' population structure, we propose a Dynamic Stochastic General Equilibrium model to evaluate how immigration affects the receiving country sustainable debt and asset prices. Our model features consumer heterogeneity, death risk, population growth, skills, labor efficiency, life cycle with periods of labor inactivity, and social security transfers. We calibrate the model using Canada data and perform a set of experiments. Specifically, we evaluate how each factor characterizing the immigrating population, factors such as immigrants age profile, fertility, or relative labor efficiency, affects the receiving country fiscal solvency. Our response variable is the change in sustainable debt resulting from a temporary immigration shock. Whatever the driver we consider, immigration seems to improve fiscal solvency. Largest changes in sustainable debt result from moves in immigrants' fertility and relative efficiency in labor.

Keywords: Immigration, Fiscal Policy, Sustainable Debt, DSGE, Labor efficiency.

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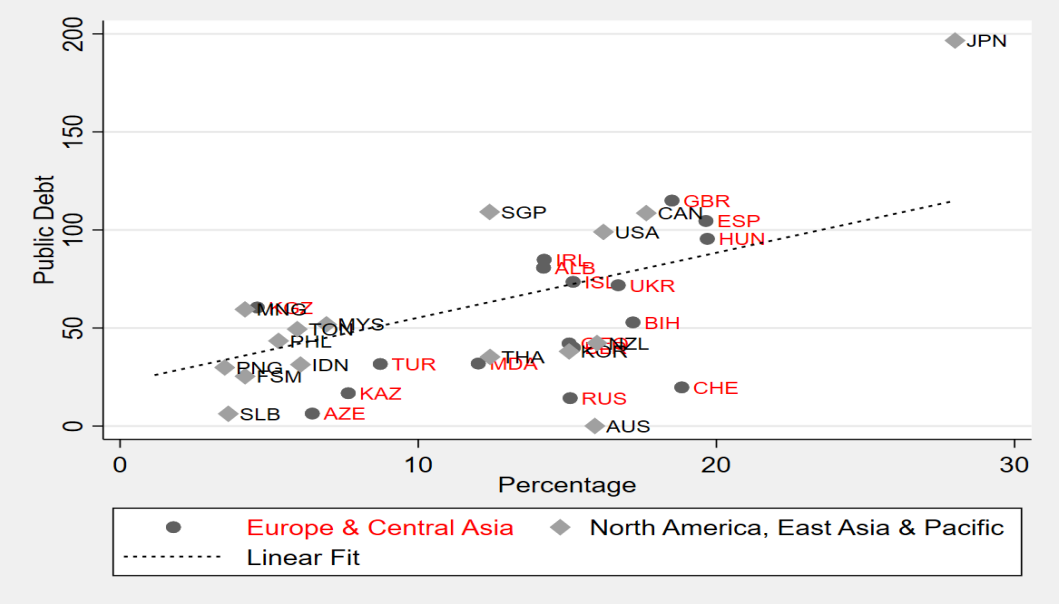
1 Introduction

Evidence of a relationship between population age profile and public debt burden in OECD countries has become increasingly apparent over the past two decades. Indeed, during this same period, the population of these countries began to age: not only has the median age of Western society increased by more than 5 years, but also the proportion of older people (who are often large recipients of public benefits) has become increasingly important. Based on World Bank Data Catalog, there appears to be a positive relationship between the level of public debt and the proportion of the population aged 65 and over.

Figure 1: Public Debt as % of GDP vs 65 + Population as % of Total Population, most recent data



Sources: From Authors based on World Bank Data Catalog.

As the scientific society became aware of the link between tax burden and demographic change, it became important to rigorously examine reform proposals that may directly or indirectly affect debt sustainability. Immigration reforms are an example of these, as they can change expectations in terms of fiscal balances. This issue has received increasing attention, in both public and scientific debates. In Denmark 2001 elections, immigration policy was a key topic; an argument to limit immigration was the resulting tax burden. Previous studies find that the fiscal contribution of the immigrant population as a whole is quite small (Rowthorn (2008)). However, once age and education are taken into account, young and highly skilled immigrants generate significant net contributions, while low-skilled retirees give rise to significant costs (Lee and Miller (2000) and Storesletten (2000)). Indeed, as might be expected, when immigrants enter their working lives, they make a net contribution to retirees through tax payments. But these

immigrants will eventually retire and receive pensions, the present value of which might or might not outweight their positive contribution during the work period. This raises a concern about the long-run sustainability of policies that rely on skilled immigration to close short-to-medium-run fiscal deficits.

Most studies dealing with the macroeconomic effects of immigration do not directly address sustainable debt, as defined by D'Erasmo et al. (2016)-put simply, sustainable debt is that initial level of debt that is covered by government present discounted value of all primary balances. Moreover, they focus on the positive aspects of immigration. The contribution of our study is therefore diverse. First of all, it is original in the sense that it addresses the question of debt sustainability, following an immigration shock. In addition, it proposes an elaborate theoretical model. Indeed, previous immigration studies only include heterogeneity in the production of agents (skilled or unskilled) and do not take into account the fact that the age structure of immigrants have different macroeconomic consequences, especially when looking at the effects on public finances through social security. So we propose a Dynamic Stochastic General Equilibrium model with period of inactivity, working period and retirement. We provide a theoretical assessment of the implications of immigration on sustainable debt and asset prices. Following Conesa and Garriga (2008), we incorporate efficiency of work that decreases with age. We also consider that work efficiency differs by immigration status. The study by Krieger (2004) showed the importance of considering fertility when studying the macroeconomic effects of immigration; we moreover assume that immigrants fertility is higher. Our model is a modified version of D'Erasmo et al. (2016) study on sustainable debt, with different dimensions of agents' heterogeneity.

Using Canada aggregate data on population structure and skill distribution, we calibrate the status-quo economy that has no immigrants, to simulate the pre-shock equilibrium level of sustainable debt. To capture the impact of immigration on sustainable debt, we compare two economies, one which starts with residents only and the other one with immigrants with specific characteristics. To do so, we identify the factors through which immigration modifies the baseline economy and consider impulse responses from each of these factors, holding the remaining ones to what Canada immigration facts suggest. We run a set of experiments in which, a high skill immigrant is relatively less efficient than a high skill resident; we assume equal efficiency between low skill immigrants and low skill residents.[1]

Overall, our results suggest that immigration improves fiscal solvency. Not surprisingly, the impact of labor efficiency is the highest: the more efficient immigrants are, the more the host country can produce, the more revenues the government can make. Less obvious, our results suggest that there is an optimal level of efficiency to expect from

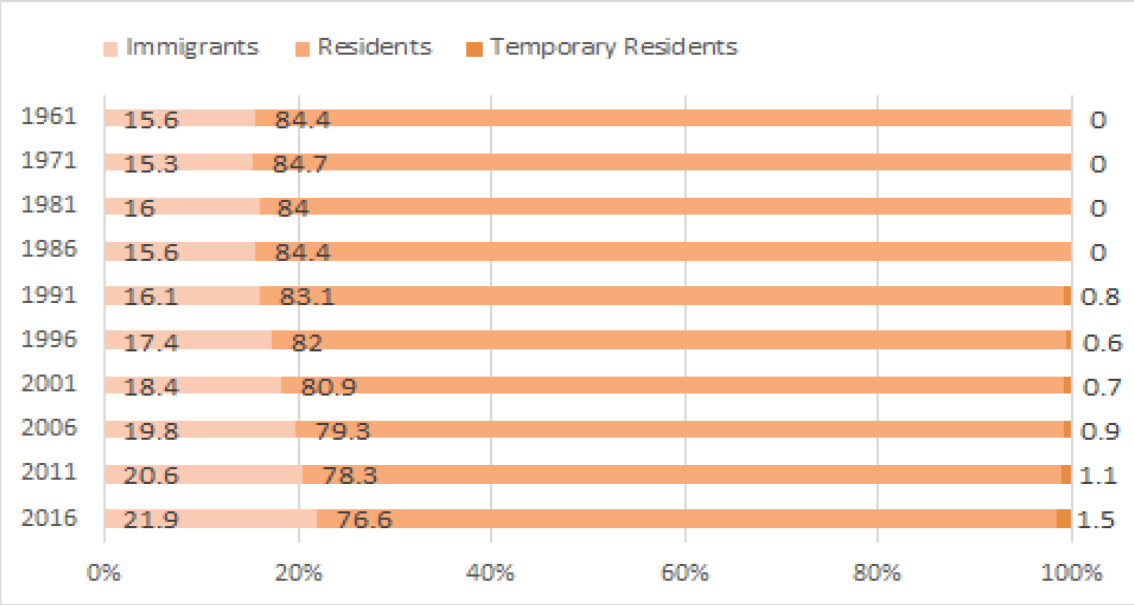
[1]In Canada, credentials obtained abroad are generally underscored. High skill immigrants undergo some training before entering the labor market.

immigrants. This is because at some point, if the taxation schedule is not appropriate, the equality in wealth distribution is jeopardized by the amount of wealth immigrants can make. The age structure of the immigrating population also matters first because taxation on labor occurs during the working life. We also find that fiscal solvency is positively affected by the share of kids in the overall immigrating population. The first reason is that kids don't affect government primary balances while they are kids. Indeed, they consume all the transfers they receive from the government. In addition, they have high probabilities to survive until the age they become productive through their entry in the labor market.

The positive impact of immigration on fiscal solvency is mostly driven by a positive change in public bond price and a slight increase of period-by period primary balances. Because the steady state share of the population that saves is higher, demand for public bond increases, therefore, public bond price increases, improving government fiscal solvency. We should note that these are results for a one-shot immigration shock. However, elaborate immigration policies will plausibly ensure a continuous influx of young immigrants that will keep supporting the previous generation. Our results are therefore lower bounds of immigration effects.

The remaining of the paper is structured as follows. In section 2 , we present the structural model used as framework to study immigration and sustainable debt. Section 3 is dedicated to the calibration of the economy with no immigrants; section 4 is dedicated to the calibration of the modified economy with immigrants. In section 5, we perform the set of our quantitative exercises and section 6 concludes.

Figure 2: A look at the evolution in the share of newly immigrants into Canada



Sources. From Authors based on Data from Statistics Canada.

2 Immigration and Sustainable debt: A Framework

Our framework nests the dynamic equilibrium model of d'Erasmo et al (2016) and Conesa & Garriga (2008). The main components are households' heterogeneity, life cycle with survival risk, efficient units of labor, fertility and social security through retirement pensions. Households' heterogeneity is captured by skill differences which are modeled through differentials in efficiency units of labor.[2]

The economy starts at date t_0. We consider a competitive equilibrium with heterogeneous households, a representative firm and the government. The households consume and supply labor to the firm. The firm produces using the labor supplied by households. The government redistributes the wealth levied through taxes on households and firms. Later, we discuss the objectives of each of these agents more extensively.

The main objective of the paper is to assess how the entry of immigrants affects the sustainability of the debt in the host country. Therefore, we will begin by studying the status quo which is the economy with no immigrants. This setup will also serve to calibrate the model for our quantitative exercises. We will then show how the framework is modified, once immigrants are added to the picture.

2.1 Baseline economy

In this section, I will discuss in detail the economy with no immigrant. We introduce immigrants in section 2.3.1 further below.

2.1.1 *Households*

The baseline economy consists of overlapping generations of resident consumers with stochastic lifetimes that last up to I years. We introduce immigrants in Section ... further. We denote the conditional probability of survival from age i to $i+1$ by ϕ_i. The unconditional probability of living until age i is then given by $s_i = \prod_{j=1}^{i-1} \phi_j$.
Denoting γ_r, the population growth rate, the measure of households of age i at time t $\mu_{i,t}$, is computed as:

$$\mu_{i,t} = \phi_{i-1}\mu_{i-1,t-1} \quad with \quad \mu_{1,t} = (1+\gamma_r)\mu_{1,t-1}. \tag{1}$$

(3.1) implies that $\mu_{i,t} = (1+\gamma_r)\mu_{i,t-1}$ for any i. As the economy begins at time t_0, the number of resident consumers is given by $N^R_{t_0} = \sum_i^J \mu_{i,t_0}$.

Workers enter the labor market at age i_w and retire at age i_r. They have 1 unit of time to split between work and leisure. They differ in the skill content of their labor

[2]It is also possible to inherently distinguish between skilled and unskilled labor, so that they enter differently in the production function. In such setup, there would be a specific price for each type of labor. We abstract from this specificity for now.

hours, which varies by their work experience, as captured by their age, i, and by their skill level $j \in \{L, H\}$. A fraction Φ of residents are high skilled (H) and the remainder is low skilled (L). Let ϵ_{ij} denote the effective labor supply per unit of time in efficiency units. Note that skill types differ in the age profiles of their productivity.[3]

Households consume at every age. They have access to the credit market only after they enter the labor market, meaning at ages above i_w. Public bond d is the only asset.

Consumption and wealth accumulation are funded by resources drawn from different sources: earnings from labor during active life, return on wealth, and government transfers made of lump sum transfers e_t to all agents, and pensions p_t paid to retirees. Consumers also pay taxes to the government, specifically a tax on consumption τ_c and a tax on labor income τ_L.

In period t, the utility of a consumer with skill j, born in period s ($t - I < s <= t$) whose age is i=t-s, is given by $u(c_{i,j,t}, l_{i,j,t})$. Agents choose consumption stream c and labor supply l (rented to firms), as well as wealth transfer d to maximize lifetime utility subject to budget constraints. In our setup with life cycle and periods of inactivity and activity, households face three budget constraints throughout their lifetime. Before labor market entry, the consumer is inactive and does not have access to the credit market.

$$U = \sum_{i=1}^{i=I} s_i \beta^i u(c_{i,j,t}, 1 - l_{i,j,t}) \tag{2}$$

The lifelong utility is the discounted sum of utility at each age i, the discount rate being β. Age i utility is also weighted by the survival rate s_i, which is the probability that the consumer reaches age i. Before working, meaning for i satisfying $i < i_w$:

$$(1 + \tau_c) c_{i,j,t} = e_t \tag{3}$$

The consumer pays a consumption tax τ_c. Before they are active workers, consumers are excluded from the financial market, so that the only revenue is the lump sum transfer from the government e. During the working life, meaning for i satisfying $i_w \leq i < i_r$:

$$(1 + \tau_c) c_{i,j,t} + (1 + \gamma) q_t d_{i,j,t+1} = (1 - \tau_L) w_t \epsilon_{i,j} l_{i,j,t} + d_{i,j,t} + e_t \tag{4}$$

Active workers accumulate wealth through public debt purchases d; the factor $(1 + \gamma)$ results from imposing balanced growth, with γ the growth rate of production.[4] The consumer of age i and skill j, that allocates $l_{i,j,t}$ units of time to work, earns $(1 - \tau_L) w_t \epsilon_{i,j} l_{i,j,t}$ after labor $\tau_L w_t \epsilon_{i,j} l_{i,j,t}$ taxes are levied.

[3] This fraction is the same at each period because survival probability is only age-dependent.

[4] Production growth rate is a function of the population growth rate.

After retirement, meaning for i such that $i_r \leq i \leq I$

$$(1+\tau_c)c_{i,j,t} + (1+\gamma)q_t d_{i,j,t+1} = p_t + d_{i,j,t} + e_t \tag{5}$$

Unlike active workers, retirees do not earn labor revenue; instead, they receive pensions p from the government.

2.1.2 *Firms*

The representative firm rents labor from households and produces y_t.[5]

$$y_t = f(l_t) \tag{6}$$

2.1.3 *Government*

The government intervenes in the economy through outlays, taxes and public indebtedness. More specifically, revenues come from consumption taxes (τ_c), labor income taxes (τ_l) and debt issuance (d_t).[6] These revenues are allocated to public consumption g, lump-sum transfers to all consumers e and pension paid to retirees p, all taken exogeneously. In other words, the government primary balance pb_t, which is equal to revenues net of expenses, is funded by the change in debt net of debt service. We assume the government is committed to repay its debt, and thus it must satisfy the following sequence of budget constraints for t = t_0, ...,∞. We denote L_t aggregate labor, C_t aggregate consumption, and P_t, aggregate pension paid.

$$pb_t = \tau_C C_t + \tau_L w_t L_t - (g_t + E_t + P_t) \tag{7}$$

$$pb_t = d_t - (1+\gamma)q_t d_{t+1} \tag{8}$$

[5]We abstract from capital. This is not essential for the purpose of the study, because we already have public debt as asset. Moreover, it is overall accepted that there is a rate to which capital can be substituted with labor.

[6]We consider tax on consumption and labor income as in D'Erasmo et al. (2016) but exclude tax on savings because we chose to keep savings free of distortions. Consumption on overall income would have achieved the same.

With:

$$L_t = \sum_{s=t-I}^{s=t} (\Phi \mu_{t-s,t} \epsilon_{t-s,H,t} l_{t-s,H,t} + (1-\Phi)\mu_{t-s,t}\epsilon_{t-s,L,t} l_{t-s,L,t})$$

$$C_t = \sum_{s=t-I}^{s=t} (\Phi \mu_{t-s,t} c_{t-s,H,t} + (1-\Phi)\mu_{t-s,t} c_{t-s,L,t})$$

$$E_t = (\sum_{s=t-I}^{s=t} \mu_{t-s,t}) e(t)$$

$$P_t = (\sum_{s=t-i_r}^{s=t} \mu_{t-s,t}) p(t)$$

Following d'Erasmo et al (2016), **public debt is sustainable** if the Intertemporal Government Budget Constraint (**IGBC**) holds. The IGBC condition is equivalent to the government satisfying a No-ponzi game condition: the discounted value of the stream of primary fiscal balances equals the initial public debt d_0. When the model is worked in shares of GDP, y_t, which will be the case for model calibration, the IGBC in shares of GDP writes:

$$\frac{d_{t_0}}{y_{t_0-1}} = \frac{y_{t_0+1}}{y_{t_0}} \left(\frac{pb_{t_0}}{y_{t_0}} + \sum_{t=1}^{\infty} \left(\left[\prod_{i=0}^{t-1} v_i\right] \frac{pb_t}{y_t}\right)\right) \quad with \quad v_i = (1+\gamma)\frac{y_{i+1}}{y_i} \tag{9}$$

2.2 Equilibrium

In our quantitative analysis, we study the recursive competitive equilibrium of the economy defined formally below. **Proposition** *Given preferences, initial population structure* $(N^R_{t_0}, \{\mu_{i,t_0}\}_{i=1}^{I}, \Phi, \gamma_r)$ *and taxation schedule* (τ_C,τ_L)*, an equilibrium is a collection of allocations for high skill resident* $\{c_{i,H,t}, l_{i,H,t}, d_{i,H,t+1}\}_{i=1..I,t=t_0...+\infty}$*, low skill resident* $\{c_{i,L,t}, l_{i,L,t}, d_{i,L,t+1}\}_{i=1..I,t=t_0...+\infty}$*, a demand schedule from the firm* $\{l_t\}_{t=t_0}^{+\infty}$*, a government policy* $\{E_t, P_t, g_t\ d_{t+1}\}_{t=t_0}^{+\infty}$*, and a price system* $Q=(q_t)_{t=t_0}^{+\infty}$ *such that the following is satisfied:*

i Optimality: given the price system Q, consumers' utility and firms' profit are maximized.[7]

ii Feasibility: the market for good, the market for labor, and the market for public debt clear for all t:

$$(Good)\ Y_t = C_t + g_t \tag{10}$$

$$(Labor)\ l_t = L_t \tag{11}$$

$$(Public\ Debt)\ d_t = \sum_{s=t-I}^{s=t} (\Phi \mu_{t-s,t} d_{t-s,H,t} + (1-\Phi)\mu_{t-s,t} d_{t-s,L,t}). \tag{12}$$

[7]Optimality conditions are provided in appendix.

iii The government's policy satisfies its budget constraint:

$$pb_t = \tau_C C_t + \tau_L w_t L_t - (g_t + E_t + P_t) = d_t - (1 + \gamma_r) q_t d_{t+1}.$$

The next section discusses how the entry of immigrants modified our baseline economy.

2.3 Adding immigrants to the economy

The main objective is to evaluate how the entry of immigrants affects sustainable debt. It is therefore critical to study the population dynamics for the quantitative part. We add upper-scripts to differentiate immigrants (i) to residents (r).

2.3.1 *Demographic structure and dynamics*

$(N^R_{t_0}, \{\mu_{i,t_0}\}^I_{i=1}, \Phi, \gamma_r)$ characterizes the baseline population structure. Let's consider the modified economy where there is a mass $N^I_{t_0}$ of immigrants with a proportion λ that are skilled. $\gamma_{r,i}$ is the adjusted population growth rate so that $(N^R_{t_0} + N^I_{t_0}, \{\mu^R_{i,t_0} + \mu^I_{i,t_0}\}^I_{i=1}, \Phi\lambda, \gamma_{r,i})$ characterizes the new demographic structure. The initial structure evolves over time due to death probability and different fertility rates between immigrants and residents. We classify all newly born consumers are residents. As Woldmicael and Roderic (2010) show, fertility is on average higher for immigrants entering Canada, as compared to canadian-born, so that $\gamma_{r,i} > \gamma_r$ until all reproductive immigrants present at t_0 disappear. Then $\gamma_{r,i} = \gamma_r$.

The share of immigrant households in total population N_t, is given by $\eta_t = \frac{N^I_t}{N_t}$. The share of residents in total population N_t, is given by $1 - \eta_t = \frac{N^R_t}{N_t}$. Since survival rate is the same among resident and immigrant consumers, having all newly born as residents implies that the growth rate of the resident population is the same as the growth rate of the total population. Therefore the share of the residing population remains constant over time: $1 - \eta_t = 1 - \eta_{t-1}$ so $\quad \eta_t = \eta$.

Next, we characterize the immigrant consumer taking the resident consumer as reference.

2.3.2 *Differences between residents and immigrants*

In addition to fertility differentials, the main difference between residents and immigrants relates to labor efficiency. In the quantitative part, most of our experiments are performed with the assumption that high skilled residents are more efficient than high skilled immigrants, while low skilled immigrants as efficient as low skilled residents.[8] We also assess the sensitivity of sustainable debt to the parameter capturing the relative labor efficiency of low skill immigrants.

[8] High skill workers will generally go through some training before integrating the labor market.

So:

$$\epsilon^r_{i,H} > \epsilon^i_{i,H} \quad we \quad set \quad \epsilon^i_{i,H} = \kappa\epsilon^r_{i,H} \quad with \quad \kappa < 1$$
$$\epsilon^i_{i,L} = \psi\epsilon^r_{i,L} \quad we \quad set \quad \psi = 1 \quad in \quad most \quad experiments.$$

In the next section, we discuss the calibration of the baseline economy.

3 Calibration of Baseline Economy to Canada

3.1 Demographics

We set the length of life to three periods, so the household spends one period in each phase of life: childhood, employment and retirement. Thus, $i_w = 2$ and $i_r = 3$. One period would be 30 years so that households are inactive from 0 to 30 years old , they work from 30 and retire at 60 years of age.[9] Survival probabilities ϕ_i are taken from Bell and Miller (2005) and aggregated to match the age profile in our setup. This implies survival probability of 0.996 from age 1 to 2, of 0.953 from age 2 to 3 and of 0 at age 3.

From Woldemicael and Beaujot (2010), we set the average number of residents' children to 1.59.[10] Assuming that only age 2 agents reproduce, $\gamma_r = 0.58$ as shown below:[11]

$$\mu_{1,t} = 1.59\mu_{2,t}$$
$$\mu_{2,t} = \phi_1\mu_{1,t-1}$$
$$\mu_{1,t} = 1.59\phi_1\mu_{1,t-1}$$
$$\gamma_r = \frac{\mu_{1,t} - \mu_{1,t-1}}{\mu_{1,t-1}} = 1.59\phi_1 - 1 = 0.58$$

Using Population data from Canada in 2018, we set the initial population age profile to $(\mu_{1,t_0}, \mu_{2,t_0}, \mu_{3,t_0}) = (0.16, 0.66, 0.17)$, which implies that $N^R_{t_0} = 1$.[12] We set the share of high skill workers to 0.4.[13]

[9] The assumption of three periods implies that the working life and retirement are of the same length. Thus, the dependency ratio implied by our model would be larger than in reality. Having longer working life would improve public finance and thus debt sustainable irrespective of the presence of immigrants. It could be interesting to assess how sensitive debt sustainability is to the length of working life.

[10] Woldemicael and Beaujot (2010) estimate to 1.76 the number of children from 35-44 years old foreign born women in 2002, and to 1.59 the same for canadian-born women of the same ages, for the same year.

[11] One period is equivalent to 30 years in this model; this explains the high implied population growth.

[12] The share by age groups are taken from Annual Demographic Estimates: Canada, Provinces and Territories, 2018, Statistics Canada, Demography Division.

[13] Figure 1, OECD (2004), for 2001.

3.2 Endowments

Efficient units of labor is households' endowments. Our efficiency units of labor are based on Hansen (1993)'s estimates who provide efficiency units for a finer age profile. Thus, we average to get efficient units of labor in our case, assuming that the values provided are those of a highly skilled worker. Since workers are active for only one period that covers ages from 30 to 60, we get $\epsilon^R_{2,H} = 1.97$. To get efficient units of labor for low skilled workers, we use hourly wage by union coverage status: workers with no union coverage earn 83% of the hourly wage of unionized workers. Thus, $\epsilon^R_{2,L} = 1.97 \times 0.83 = 1.64$. For Consumers also receive lump-sum transfers e_t at all ages; pensions p_t are paid to retirees. We impose that both type of transfers are fixed over time. We provide their values below, as we discuss the calibration of the public sector.

3.3 Government

Taxes are taken directly from OECD releases: we set consumption tax to $\tau_C = 0.05$ and labor income tax to $\tau_L = 0.19$.[14]

To compute variables related the government, we take some directly from data provided by Statistics Canada (the fourth quarter of 2019). For government final consumption g, we impute $\frac{g}{Y} = 0.20$.[15] Primary balance as share of GDP is set to $\frac{Pb}{Y} = 0.01$.[16] To get the implied value of total transfers to households $\frac{E+P}{Y}$, we need $\frac{C}{Y}$ which is obtained from the equilibrium on the market for final good $\frac{C}{Y} = 1 - \frac{g}{Y} = 0.8$, so that $\frac{E+P}{Y} = 0.04$. From OECD stats, $\frac{P}{Y} = 0.048$ which exceeds the value that our model would imply, thus, we set $\frac{P}{Y}$ to 0.03 and $\frac{E}{Y}$ to 0.01.[17] Turning to the value of transfers per consumer consumer, since $N^R_{t_0} = 1$, e=E; e is fixed over time, so $\frac{E}{Y}$ changes over time. Pensions are paid to retirees only, so $\frac{p}{Y} = \frac{\frac{P}{Y}}{\mu_{t_0}} = 0.18$.

3.4 Functional Forms

Households preferences are assumed to take the following form: $u(c, l) = logc + log(1 - l)$ which implies a relative risk aversion of 1.

So, for j={L,H}, $U_j = \sum_{i=1}^{i=3} s_i \beta^i (log(c_{i,j,t}) + log(1 - l_{i,j,t}))$.

As of technology, we work with the following production function $f(L_t) = L_t$ so that wage is equal to 1 at each period.

[14] OECD Stats, Table I.6. All-in average personal income tax rates at average wage by family type.

[15] Authors' computations based on Statistics Canada. Table 36-10-0222-01 Gross domestic product, expenditure-based, provincial and territorial, annual (x 1,000,000) is used to compute consumption as shares of GDP.

[16] Authors' computations based on Statistics Canada. Table 36-10-0477-01 Revenue, expenditure and budgetary balance - General governments (x 1,000,000).

[17] OECD (2021), Pension spending (indicator). doi: 10.1787/a041f4ef-en (Accessed on 30 June 2021).

The table below resumes how we calibrated the status-quo economy (without immigrants).

Calibration of baseline economy

Parameters	Value	Source
γ_r	0.58	Authors computations based on Woldmicael et al. (2010)
i_w	2	From authors
i_r	3	From authors
$(\mu^R_{1,t_0}, \mu^R_{2,t_0}, \mu^R_{3,t_0})$	(0.16,0.66,0.17)	Canada population Data (2018)
(ϕ_1, ϕ_2, ϕ_3)	(0.99, 0.95, 0)	Averages based on estimates from Bell and Miller (2005)
Φ	0.4	Figure 1, OECD (2004), for 2001
$\epsilon^R_{2,H}$	1.97	Hansen (1993)
$\epsilon^R_{2,L}$	1.64	Hourly wage ratio (no union coverage to union coverage), 2019
(τ_C, τ_L)	(0.05,0.18)	OECD Stats
β	0.998	D'Erasmo et al. (2016)

Notes. We use the hourly wage ratio between workers uncovered by a union and workers covered by a union to capture the relative efficiency between high skill and low skill workers.

Next, we show how key parameters of the baseline economy are affected by the entry of immigrants.

4 Economy with immigrants

Some components of the population structure are modified. At t_0, the population structure is now $(N^R_{t_0} + N^I_{t_0}, \{\mu^R_{i,t_0} + \mu^I_{i,t_0}\}^I_{i=1}, \Phi, \lambda, \gamma_{r,i})$. Based on data from Statistics Canada and as shown on Figure 2, newly arrived immigrants represent approximately 22% of the whole population of Canada. Thus

$$N^I_{t_0} = 0.22(N^I_{t_0} + N^R_{t_0})$$
$$N^I_{t_0} = 0.28$$

. The age profile of immigrants in 2011 implies that $(\mu^I_{1,t_0}, \mu^I_{2,t_0}, \mu^I_{3,t_0}) = (0.33, 0.64, 0.03)$.[18] As of the share of highly skilled immigrants, following results of King (2009), we set λ to 0.41 .[19] Based on Woldemicael and Beaujot (2010), the number of children from 35-44 years old foreign born women in 2002 is 1.76. Thus, the growth rate at t_0 is given by

$$\gamma_{r,i} = \frac{1.59\mu^R_{2,t_0} + 1.76\mu^I_{2,t_0} - \mu^R_{1,t_0} - \mu^I_{1,t_0}}{\mu^R_{1,t_0} + \mu^I_{1,t_0}}$$
$$\gamma_{r,i} = 3.43$$

[18] Sources. Statistics Canada, Catalogue no. 99-010-X2011001 ISBN: 978-1-100-22197-7.
[19] Martin Prosperity Institute REF. 2009-WPONT-012.

From t_0+1 on, there is no age 2 immigrants because all newly born are residents; thus $\gamma_{r,i}=0.58$.

To get the relative labor efficient units between immigrants and residents, we use weekly wages of university educated new immigrants and compare it to their resident counterparts. In 2006, university graduated immigrants earned on average 88% of the weekly wage of university graduated canadian born.[20] So $\epsilon^I_{2,H}$=1.97×0.88 = 1.75.

Calibration of modified economy

Parameters	**Value**	**Source**
$\gamma_{r,i}$	(3.43; 0.58)	Authors computations based on Woldmicael et al. (2010)
i_w	2	From authors
i_r	3	From authors
$(\mu^R_{1,t_0},\mu^R_{2,t_0},\mu^R_{3,t_0})$	(1, 0.16, 0.66, 0.17)	Canada population Data
$(\mu^I_{1,t_0},\mu^I_{2,t_0},\mu^I_{3,t_0})$	(0.33, 0.64, 0.03)	Canada population Data (2011)
N^I_{1,t_0}	0.28	Computations based on Figure 2, 2016
(ϕ_1,ϕ_2,ϕ_3)	(0.99, 0.95, 0)	Averages based on estimates from Bell and Miller (2005)
λ	0.41	King (2009), value for 2006
$\epsilon^I_{2,H}$	1.75	Hansen (1993) and Statistics Canada, 2006
$\epsilon^I_{2,L}$	1.64	Weekly wage university graduates, 2006
(τ_C,τ_L)	(0.05,0.18)	OECD Stats
β	0.998	D'Erasmo et al. (2016)

Notes. We use the hourly wage ratio between workers uncovered by a union and workers covered by a union to capture the relative efficiency between highly skilled and low skill workers.

We perform a set of exercises to assess the impact of receiving immigrants on government fiscal solvency. The parameters that capture how immigration affects the economy are $(\lambda,\epsilon^I_{2,H},\epsilon^I_{2,L},1-\eta,\gamma_{r,i},(\mu^I_{1,t_0},\mu^I_{2,t_0},\mu^I_{3,t_0}))$. In all the experiments we conduct below, the entry of immigrants is a temporary, meaning that in the modified economy, a mass $N^I_{t_0}$ of immigrants enter at date t_0. Also, all newly born are residents.

In the next section, we provide our quantitative results on the effect of immigration on sustainable debt.

5 Quantitative assessment of the effect of immigration on sustainable debt

Firstly, we assess how the share of immigrants in the whole population affects debt sustainability.

[20] Source(s): Canadian censuses of 1981, 1991, 2001 and 2006 20% files; U.S. censuses of 1980, 1990, and 2000 IPUMS 5% files and 2005 American Community Survey IPUMS 1% file.

5.1 Sensitivity to the total share of immigrants $1-\eta$

Here, we are assessing how fiscal solvency is affected by the total share of immigrants in the population. Therefore, we compute sustainable debt as a function of that share. We use Canada immigration data to set a value to the remaining parameters $(\lambda, \epsilon^I_{2,H}, \epsilon^I_{2,L}, \gamma_{r,i}, (\mu^I_{1,t_0}, \mu^I_{2,t_0}, \mu^I_{3,t_0}))$ = $(0.41, 1.75, 1.64, (3.34, 0.58), (0.33, 0.64, 0.03))$. Using these values, we get the sustainable debt, which is the present discounted value of inter-temporal primary balances. With status-quo sustainable debt as baseline value (without immigrants), we compute the percentage change: the variable along the vertical axis. So the curve on figure 3 maps values of $1-\eta$ into the percentage change of sustainable debt: the equilibrium present discounted value of the primary fiscal balance for the modified economy (with immigration) relative to the simulated value for the baseline economy (without immigration).

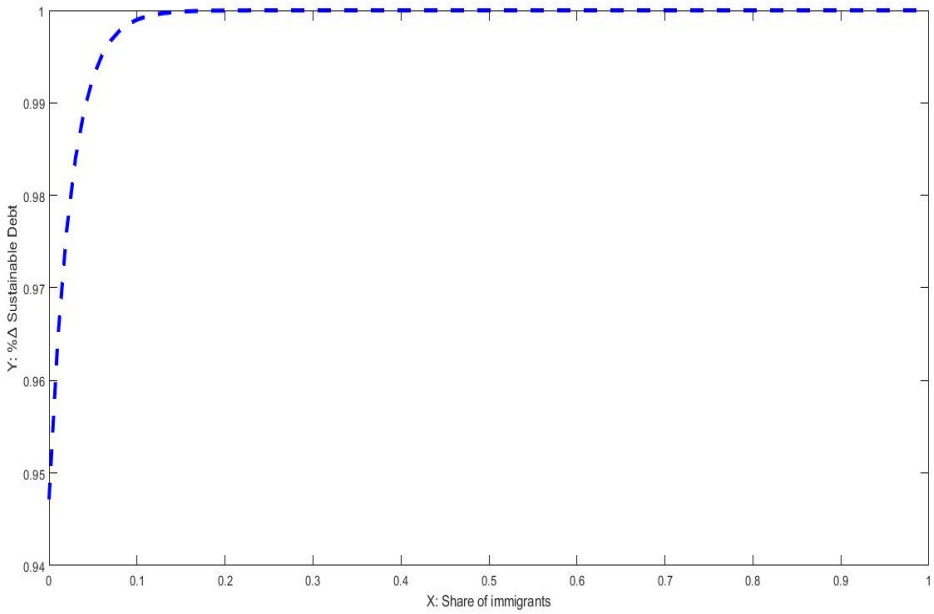


Figure 3: Change in sustainable debt as a function of immigration intensity

We see that the entry of immigrants increases sustainable debt, since the values on the vertical axis are all positive. There is a sharp increase in sustainable debt for low values of the mass of entrants, then the change stagnates to 1, which means that sustainable debt has doubled. This happens once we hit a share of immigrants of approximately 15%. This result was predictable because what matters the most is the age profile of immigrants, which is fixed to the age profile of canadian immigrants. Indeed, each immigrant pays consumption tax at all ages, pays income tax only during working life and retirement, receives lumpsum transfers at all ages and pensions during retirement. When immigrants

enter, it creates a shock to the status quo economy and primary balance would be positively affected if the share of active immigrants is high enough. It is important to note immigration data from Canada features a high fraction of immigrants in their working life. It is also interesting to note that each year, the share of immigrants in the whole population is approximetely 20%.

5.2 Sensitivity to age profile of immigrants: fractions taken by pairs $(\mu^I_{1,t_0}, \mu^I_{3,t_0})$, $(\mu^I_{2,t_0}, \mu^I_{3,t_0})$

To better capture the influence of the age profile, it takes to consider fractions of the population at least by pairs. Indeed these fractions are dependent from each other since $(\mu^I_{1,t_0} + \mu^I_{2,t_0} + \mu^I_{3,t_0} = 1)$. So, we compute sustainable debt as a function of two ages. As before, we use Canada immigration data provided in table 2.2 to set a value to the remaining parameters $(\lambda, \epsilon^I_{2,H}, \epsilon^I_{2,L}, 1-\eta, \gamma_{r,i}) = (0.41, 1.75, 1.64, 0.28, (3.43, 0.58))$. The curve on the left of figure 4 maps $(\mu^I_{1,t_0}, \mu^I_{3,t_0})$ to the percentage change of sustainable debt, and the second graph maps changes in $(\mu^I_{2,t_0}, \mu^I_{3,t_0})$ to the percentage change in sustainable debt. Thus, the younger the immigrating population, the more the the public finances of hosting country are improved.

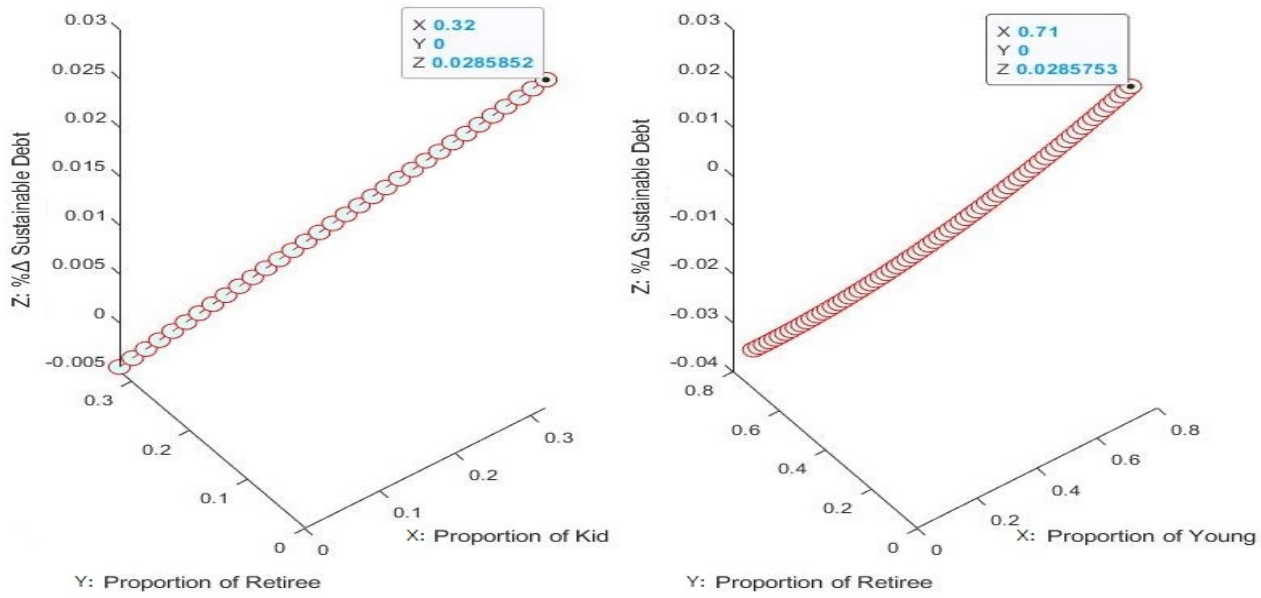


Figure 4: Change in sustainable debt as a function of immigrants' age fractions taken by pairs

5.3 Sensitivity to immigrants' skill $(\epsilon^I_{2,H})$

The focus here is on the impact on fiscal solvency of the relative efficiency in labor of immigrants. We set the remaining parameters from data $(\lambda, 1-\eta, \epsilon^I_{2,L}, \gamma_{r,i}, (\mu^I_{1,t_0}, \mu^I_{2,t_0}, \mu^I_{3,t_0})) = (0.41, 0.28, 1.64, (3.43; 0.58); (0.33, 0.64, 0.03))$. Figure 5, maps $\epsilon^I_{2,H}$ to the change in sustainable debt once immigration has happened. Thus, the more immigrants are efficient in labor, the better debt sustainability would be after they enter the country. However, above a certain threshold, the gain in debt sustainability decreases with the labor efficiency of immigrants. In fact, consumption of active workers is a function of their labor efficiency. However, tax on labor income is flat which might create distribution issues. Indeed, consumption of kids are fully funded by government transfers and consumption retirees are partially funded by pensions paid by the government. At some point, public revenues does not increase as much as immigrants wealth does, leading to a relatively lower gain in debt sustainability.

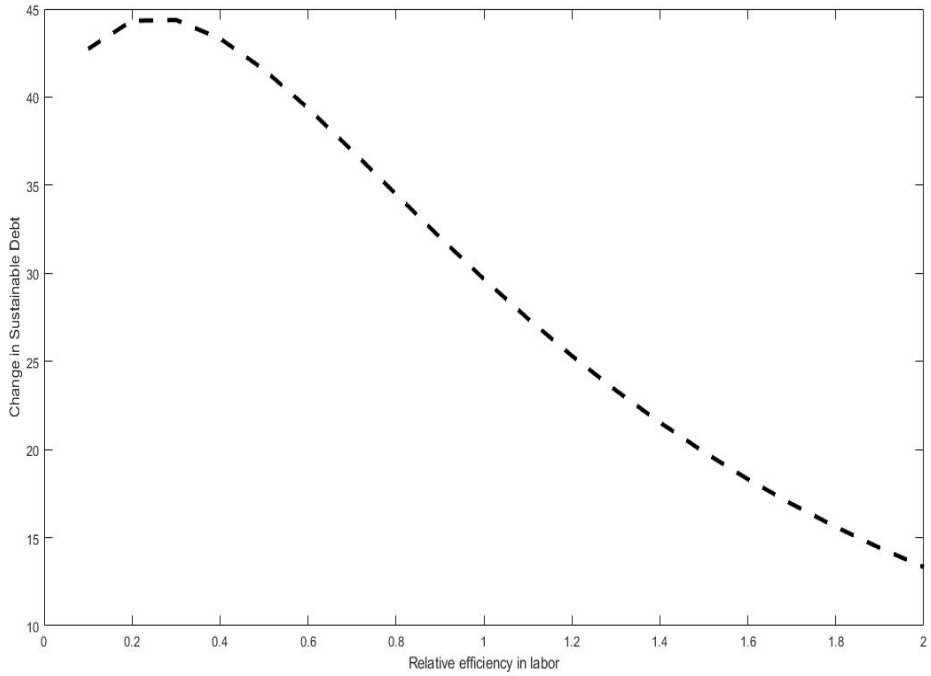


Figure 5: Change in sustainable debt as a function of immigrants' labor efficiency and type of skill intensity

5.4 The reason behind: Immigration shock, primary balance and price of public bond

From results just discussed, immigration appears to be beneficial for the host country fiscal solvency. We saw that sustainable debt is the discounted present value of all primary balances; it is equivalent to the no-ponzi game condition for the government. Therefore if one can measure the effect of immigration on asset price (inverse of gross return) and on each period primary balance, it will be straightforward to grab the change in sustainable debt. The figure below addresses this point. Immigration parameters are set like this $(\lambda, \epsilon^I_{2,H}, \epsilon^I_{2,L}, 1-\eta, \gamma_{r,i}, (\mu^I_{1,t_0}, \mu^I_{2,t_0}, \mu^I_{3,t_0})) = (0.41, 1.75, 1.64, 0.28, (3.34, 0.58), (0.33, 0.64, 0.03))$. The graph on the left represents two curves, primary balance as shares of GDP from period $t = t_0 = 0$ to period t=30, for the baseline economy (without immigrants in blue) and for the modified economy (after immigration, in red). The graph on the right does the same for asset (public bond) price.

An immigration shock similar to Canada yearly immigration will initially deteriorate government primary balance. It is not clear on the graph, but at some point, primary balance becomes positive and larger than what it would have been in the absence of immigrants. For asset price, it takes some years before the equilibrium price of public bond differs from what it would have been without immigrants; we can see that the gap is pretty large. Both the delay and the direction of the change in the asset price are consistent with what the model predicts. In fact, since the pool of immigrants is mostly made of active workers, the immigration shock increases significantly the share of households that are savers. With more savings, the demand for public bonds increases, leading to a higher price. The delay is consistent with the fact that it takes some time for assets to accumulate, and for the debt stock to significantly change.

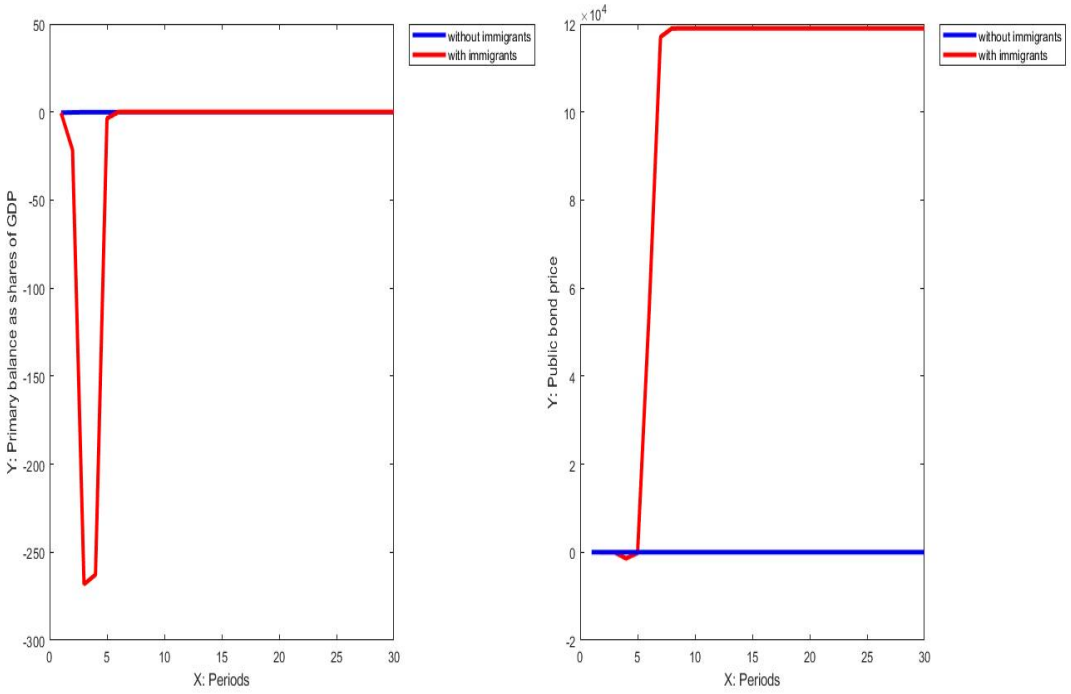


Figure 6: Primary balance and public bond price over time

6 Conclusion

Countries that receive immigrants have their population structure significantly modified. Indeed, immigration is a particular kind of demographic shock, since immigration "newborns" may have a past and characteristics that differ from natives: their skills and their propensity to reproduce are examples of those. Aging societies as Canada generally undergo massive immigration.

This paper addressed one aspect, and not the least of the various impacts that immigration has on the hosting country: fiscal solvency. When immigrants enter a country, they work, they consume, they save, all things that are beneficial for the receiving country. But they will also get ill, age, loose jobs, which will require government support. The fact that aging societies continue to receive immigrants suggests that there is more to gain that to loose. Our paper proposed a rationale to explain the positive impact of immigration. Using a DSGE model that features life cycle, death risk, skills and labor efficiency, we simulated the equilibrium effect of immigration on sustainable debt.

Sustainable debt is the level of debt that all subsequent primary balances will cover with strict equality. So the more a country is able to make fiscal surpluses, the more room it has to borrow while remaining solvable. There are different dimensions by which immigration changes the population characteristics, and whatever the aspect we consider, our results show that immigration improves fiscal solvency. The entry of immigrants, mostly made of active workers increases the share of the population that saves. Therefore, the

demand for public bonds increases driving price of public bond up. Age and skills of immigrants are characteristics that boost fiscal solvency the most. The younger the immigrating population, the better the public finances. The more efficient the immigrating population is, the better the fiscal solvency of the host country. However, with a flat tax rate on labor income, at some point, the positive impact of immigrants' labor efficiency on the fiscal solvency is reduced. The reason is that government revenues do not increase as much as immigrants' revenues. This result emphasizes that immigration may affect wealth distribution.

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